

## Definitions

Angular displacement  $\theta$  indicates the angle through which an object has rotated. It is measured in radians.

Average angular velocity  $\omega$  is angular displacement divided by the time interval over which that angular displacement occurred. It is measured in rad/s.

Instantaneous angular velocity is how fast an object is rotating at a specific moment in time.

Angular Acceleration  $\alpha$  tells how much an object's angular speed changes in one second. It is measured in rad/s per second.

Angular acceleration and centripetal acceleration are independent. Angular acceleration changes an object's rotational speed, while centripetal acceleration changes an object's direction of motion.

## Relationship between angular and linear motion

The linear displacement of a rotating object is given by  $r\theta$ , where  $r$  is the distance from the rotational axis.

The linear speed of a rotating object is given by  $v = r\omega$

The linear acceleration of a rotating object is given by  $a = r\alpha$ .

## Torque

The torque is the force on a rotating object. It is found using the formula  $\tau = Fd\sin\theta$ .

If an object is in equilibrium then it experiences a net torque equal to 0.

Net Torque Problem Solving Procedure

1. Identify the objects providing the torque
2. Calculate your clockwise torque
3. Calculate your counterclockwise torque
4. a. If the object is balanced (in equilibrium), set your torques equal to each other.
5. b. If the object is not balanced, find the net torque on the object.

## Rotational Inertia

Rotational inertia  $I$  represents an object's resistance to angular acceleration.

For a point particle, rotational inertia is  $MR^2$ , where  $M$  is the particle's mass, and  $R$  is the distance from the axis of rotation.

## Newton's Second Law for Rotation

An angular acceleration is caused by a net torque:  $\tau = I * \alpha$

Where T is Torque, I is moment of Inertia, and  $\alpha$  is angular acceleration.

### **Rotational Kinetic energy**

Objects that are spinning also have energy even if they are not moving forward at all. This rotational kinetic energy is calculated with the formula  $KE_{\text{Rot}} = \frac{1}{2}I\omega^2$ .

Rolling Objects have both Linear Kinetic Energy and Rotational Kinetic Energy

$$KE_{\text{Total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

When we use our conservation of energy, we now have to include rotational energy as well!

$$PE_i + KE_i(\text{Rotational}) + KE_i(\text{Linear}) = PE_f + KE_f(\text{Rotational}) + KE_f(\text{Linear})$$

### **Angular Momentum**

The angular momentum of a spinning object is found with the formula :  $L = I\omega$

Just like linear momentum, angular momentum must be conserved if no external torque is applied.